Solving Exercise 3.4.2.

Prove or disprove:

a) (R / S)\* = R\* / S\*

Solution:

Let’s try to disprove: let R = 0, S = 1.

Then LHS = (0/1)\* which matches all possible binary strings

and RHS = 0\*/1\* which matches binary strings that are all-zeros or all-ones or the empty string (Є). So LHS matches with certain strings (for e.g. 011) which doesn’t match with RHS.

So LHS ≠ RHS.

b) (RS / R)\*R = R(SR / R)\*

Solution:

Let’s try to disprove: assume R = 0, S = 1

LHS = (01/0)\*0 and RHS = 0(10/0)\*

[(01/0)\* = Є / (01/0) / (01/0)(01/0) / (01/0)(01/0)(01/0) /…. ]

E.g. LHS matches with 0, 00 and RHS also matches with that

E.g. LHS matches with 0100 and RHS also matches with that

E.g. LHS matches with 0101000 and RHS also matches with that ...

So let’s try to prove.

Comparing shifting rule with RHS:

LHS = (RS / R)\*R

= ( R (S / Є) )\*R

( E F )\* E

= R((S / Є) R)\* [ Shifting rule: (EF)\*E = E(FE)\* ]

E ( F E )\*

= R(SR/R)\*

= RHS; So LHS = RHS (proved)

c) prove or disprove: (RS/R)\*RS = (RR\*S)\*

**Solution:**

Let’s try to disprove first.

Let, R=0, S=1 then

LHS = (01/0)\*01

RHS = (00\*1)\*

It we take for e.g. Ɛ: it doesn’t match with LHS but matches with RHS

So, LHS ≠ RHS

Rough:

~~CLAIM: 010001 matches LHS but does not match RHS~~

0100 01

(01/0) (01/0) (01/0) 01: LHS

01 0001

00\*1 00\*1:RHS

**Variation of c) Prove or disprove: (RS/R)\**RS* = (R+S)+**

Rough: RHS = (R+S)+ = (R+S)\* (R+S) = (R+S)\* R\* RS = (R\*RS)\* R\* RS = (R/RS)\* RS = **(RS/R)\**RS = LHS***

Solution:

Let’s try to disprove first.

Let R=0, S=1 then

LHS = (01/0)\*01

RHS = (0+1)+

If we consider 01, 001, 0101, etc. we see that all of these strings match both LHS and RHS. So let’s try to prove the given equation.

LHS = (RS / R)\*RS

= (R / RS)\* RS [commutative law: E/F = F/E]

( E | F )\*

= (R\*RS)\* R\* RS [denesting rule: (E|F)\* = (E\*F)\*E\*]

(E\* F )\* E\*

= (R+S)\* R+S = (R+S)+ = RHS (proved)

d) Prove of disprove: (R/S)\*S = (R\*S)\*

Solution:

Disprove :

R =0 and S=1.

LHS = (0/1)\*1 which matches any binary string that ends with 1

RHS = (0\*1)\* which matches any repetitions of 0\*1

So a string matched by the RHS is Ɛ which is not matched by the LHS. So, the statement is disproved.

e) Prove or disprove: S(RS/S)\*R = (R+S)+

Solution:

Disprove : S = 0 and R =1 then LHS = 0(10|0)\*1 and RHS = (1+0)+

The string 01 is accepted by LHS but is not accepted by RHS. (disproved)

**Variation of e): Prove or disprove S(RS | R)\*R = (SR+)+**

**Solution:**

Let’s try to disprove first.

Let S = 0, R = 1.

So LHS = 0(10/1)\*1, RHS = (01+)+

Considering 01, 011, 0101, etc. we see that all of these strings match both LHS and RHS. So let’s try to prove.

LHS

= S ( RS | R)\* R

= S ( R | RS)\* R [commutative law]

= S (R\*RS)\* R\* R [denesting rule: (E|F)\* = (E\*F)\*E\*]

= S (R+S)\*R+

=(SR+)\*SR+ [shifting rule]

=(SR+)+ = RHS (proved)

**Problems inspired by DFA=>RE:**

Try to prove the followings:

1. 10(0/1+0)\* = 10 / 10 (0/1)\*0

[binary strings that start with 10 and end with 0]

Proof:

Let, 1 = E and 0 = F

LHS = EF(F/E+F)\*

=

RHS = 10 / 10 (0/1)\*0

= EF / EF (E/F)\*F

= EF / EF (F\*E)\* F+

=

1. (0/1+0)\*1+01 (0/1)\* = (0/1)\*101(0/1)\*

[inspired by the DFA=>RE for binary strings containing 101]

Proof:

1. (0\*1/1\*)\* = (0\*1)\*